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## *Respond or Dismiss: Interactions That May Support Loving, Bullying and Solitude in Mathematics*

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### **Abstract:**

We develop an analytical frame to support reflection on how love, bullying and solitude appear in communication in mathematics classrooms. The frame distinguishes among communication acts that are responsive or dismissive to others and identifies how the acts draw on authority to open or close dialogue. Our examination of high school students' language repertoires revealed that their communication acts influence both their development of mathematics and their interpersonal relationships. To illustrate the framework in use, we draw on transcripts from group interactions in one high school mathematics class. We consider how the particular kinds of communication acts may support and develop caring, antagonism or reclusion. The illustrative analysis illuminates the complexity of human relationships—how communication acts in mathematics classrooms intertwine personal autonomy, interpersonal positioning, and how these communication acts are intertwined with interpersonal positioning.

**Keywords:** mathematics education; love; bullying; solitude; communication acts; responsiveness; dismissiveness; authority; positioning; discourse; project work; group work

## **Réagir ou rejeter : Les interactions qui pourraient encourager l'amour, l'intimidation et l'isolement en mathématiques**

### **Résumé :**

Nous développons un cadre analytique pour soutenir la réflexion sur la manière dont l'amour, l'intimidation et l'isolement apparaissent dans la communication en cours de mathématiques. Le cadre distingue les actes de communication réactifs ou dédaigneux envers les autres et identifie la manière dont ils s'appuient sur le pouvoir d'être réceptif ou de refuser un dialogue. Notre analyse des répertoires linguistiques des lycéens a révélé que leurs actes de communication influencent à la fois leur développement des mathématiques et leurs relations interpersonnelles. Pour illustrer le cadre utilisé, nous utilisons des transcriptions d'interactions de groupe dans un cours de mathématiques de niveau secondaire. Nous examinons comment certains types de communication peuvent favoriser et développer la bienveillance, l'antagonisme ou la réclusion. L'analyse fondée sur l'illustration s'éclaire la complexité des relations humaines : la manière dont les actes de communication en cours de mathématiques entremêle l'autonomie personnelle au positionnement interpersonnel aussi qu'à la manière dont ces actes de communication sont intimement liés au positionnement interpersonnel.

**Mots clés :** éducation en mathématiques; amour; intimidation; réclusion; actes de communication réactifs ou dédaigneux; autorité; positionnement interpersonnel; discours; travail en groupes

When analyzing transcripts of Canadian 15-year-olds doing a mathematical investigation, we became captivated by the very different ways the students used their (mathematical) language to either support their mathematics learning and their relations with each other in a loving and caring way or to bully or be mean to each other. Initially, the mean interactions in our transcripts and recordings raised for us questions about the way mathematics was intertwined with the bullying. This drew our attention to similar intertwining of love and care with mathematics. And we found that another form of interaction in group work was solitude—students insulating themselves from relationship. These relationships between mathematics and ways of interacting are fundamental to students' learning experience. The love, the bullying and the moves to solitude are enacted through communication acts in mathematics classrooms.

We claim that one cannot understand communication about mathematical processes without understanding that these acts are also part of repertoires for other discourses that are intertwined with that of mathematics. This is true for students trying to understand each other, students trying to understand teachers, teachers trying to understand students, and researchers trying to understand students and teachers. Positioning theory (van Langenhove & Harré, 1999), especially as it has been elaborated by Herbel-Eisenman, Wagner, Johnson, Suh and Figueras (2015), helps us understand communication acts in classrooms and how they connect to a variety of discourses, including mathematics. A central idea of positioning theory is that any communication act can be understood in a variety of ways when connected to different storylines. Most relevant to this article, communication acts in a mathematics classroom can be seen as moving mathematics forward (or not), but they may also be seen as an articulation of love or of bullying. We say these things are "intertwined" (Andersson & Wagner, 2019) because it is not that a mathematics communication supports bullying or vice-versa. One does not necessarily dominate over the other. The mathematics gives a context in which I can bully or love someone, and in which bullying or love can provide motive for my mathematical moves.

In schools there are unequal relationships in terms of power, not only relations between teachers and students, but also among students. Some research in mathematics education has drawn attention to the way language mediates these relationships. For example, power relations are in evidence with common expressions in mathematics class such as "We have to . . ." (Herbel-Eisenmann & Wagner, 2010). Noddings (2010, p. 148) invites educators to take into consideration the "expressed needs" of students. This includes more than inferred curriculum needs, teachers' needs or others' expectations on mathematics achievement.

Our attempts to understand love and bullying better have helped us realize that love and bullying are not moments in interaction but rather patterns of interaction. Thus, we have turned our attention to communication acts that have the potential for developing the kind of patterns that may be identified as love or bullying. In particular we develop a conceptual framework for focusing on the way people respond to, dismiss, or ignore the contributions of others in the context of mathematics classroom student interaction.

The heart of our question is this: How can mathematics teachers and other educators create discourses with love in mathematics learning contexts? As a start to this question, we focus this article on the following question: How is communication among students responsive, dismissive, or reclusive in expected mathematical conversations? The focus of this article is the analytic frame we developed to answer this question. We draw on examples from a current research project to illustrate the frame and to underscore the complexity of the research question. The frame is rather dissatisfying because it focuses on in-the-moment interactions and thus avoids our underlying interest in love and bullying in mathematics, but we argue that this imperfect fit honours the complexity of these kinds of human relationships. Using our example contexts, we consider how loving and bullying may manifest in mathematical dialogue, in this particular case when students work with a mathematical investigation activity.

When we have presented the earlier stages of our work we experienced two strong reactions from our audiences. Our audiences desired something in between love and bullying in our framework, more of a continuum. Also, some were challenged by the imperfection of the analytical frame we developed. Probably all of us have wished at times for a clear way to identify love (and some of us may have had the need to identify a clear case of bullying) but we know this is not possible. Our analytical frame embraces the complexity of these very real interpretations of experience and does not draw neat lines between its categories and the experiences that motivate our interest. Nevertheless, our use of the analytical frame helps us better understand the intertwined nature of mathematics education discourses and human relationships, and how these become even more complicated in group interactions, which are especially important in classrooms informed by constructivist and socio-cultural theories. Too often, presentations of analytical frames use only clear cases to illustrate the features of the frame, an approach that unfortunately misses the complexity that is inherent in most teaching dilemmas.

There is substantial scholarly work that analyses students' conversation and/or positioning in collaborative mathematical learning settings (e.g., Ellis, 2011; Esmonde, 2009; Staples, 2007; Wood, 2013). Most of this work is focused on whether or not students are developing mathematical arguments—for example, DeJarnette and González (2015) found that “students' positioning and mathematical reasoning are intertwined and jointly support collaborative learning through work on novel tasks” (p. 378). By contrast, we want to foreground the students' ways of expressing themselves in interactions.

We begin with attention to the literature's treatment of the generally ignored concepts of love, bullying and solitude in mathematics contexts. This follows with development of our conceptual frame, and finally with examples from our research, in which we identify the communication acts and positioning of students in relation to the mathematical and relational moves.

## Love

Conceptualizations of love comprise a variety of different feelings, states, emotions and attitudes. These legitimate conceptualizations may range from pleasure and care to interpersonal

affection. The definition of love we adhere to in this article is not a romantic kind of love. In this article, we understand love as the virtue that represents kindness and compassion, as "the unselfish loyal and benevolent concern for the good of another" (Love, n.d.). We see love as being attentive to the concerns of others and being responsive to them. Hence, love is an expression of positive sentiment. Because antonyms are contingent on perspective (Visser, 2000), we identify two possible antonyms for love—*hate/antipathy*, as in "I hate mathematics" or "I dislike her," and *apathy*, as in "Mathematics is boring" or "I am not interested in him".

The word *love* has a variety of related but distinct meanings, in different contexts, but also in different languages. Many languages use multiple words to express some of the different concepts that in English are denoted as "love" (Ferry, 2013; Noddings, 2013). English, too, has multiple words associated with love.

When we searched the (prominent and English) mathematics education research journals for the words *love* and *mathematics* together, rather few articles showed up. In those we found, *love* was used in a variety of ways, most often to describe relationships with mathematics and/or with mathematics education. First, there is a theme of *loving mathematics*, here exemplified with the quote by a mathematician and teacher in New Zealand when designing learning tasks:

Mathematics is joy and exasperation. It is beauty and power. As mathematicians we love elegant solutions and the way that the same mathematics can be applied to different contexts. (Wood, 2011, p. 5)

Solomon's (2009) book on mathematical literacy and peoples' experiences of inclusiveness in mathematics education includes similar stories about mathematics at all school levels from primary school up to university level in English contexts. These accounts feature students or teachers showing love of *school mathematics*. Similarly, Solomon, Radovic and Black (2016) presented a case study of a teacher loving the physics and mathematics school subjects and her "refusal to be solely determined by macro-level structures which may position her as outside of mathematics" (p. 56). Focusing on a teacher's perspective, Hossain, Mendick and Adler (2013) showed that a student teacher's choice of teaching mathematics was about her love for the subject: "After all the subjects I looked at, I thought that I loved maths the best" (p. 45). The teacher's desire to teach was inseparable from her desire to teach mathematics: "That is one of the reasons why I would like to teach maths, because I love maths and the reason why I like maths is because I had a very good maths teacher in my secondary school." (p. 45).

We problematize the idea of loving mathematics or school mathematics with reference to Wagner and Herbel-Eisenmann's (2009), use of positioning theory, which would say that there is no actual presence of mathematics in any situation. They argued that the presence of mathematics is only ever present as mediated through people (teachers, textbooks authored by people, etc.) and texts. Thus, we ask what or whom people love when they say they love mathematics? Could it be a love for the kind of relationships they experience when doing mathematics? Love or attachment to some mathematical processes may or may not go hand in hand with loving people with whom one interacts mathematically.

There are much richer accounts in mathematics education research describing opposites of love—*hating* or *not loving* (antipathy) and feelings of *apathy* or *boredom* (e.g., Andersson, Valero, & Meaney, 2015; Brown, Brown, & Bibby, 2008; Nardi & Steward, 2003; Solomon, 2009).

Esmonde and Langer-Osuna (2013), in the context of a Grade 10 heterogeneous mathematics classroom involved in group work, showed how the figured worlds (see Holland, Lachicotte, Skinner, & Cain, 1998) of friendship and romance allowed disadvantaged students to position themselves more strongly in a mathematical figured world. In one of their examples, a student (Dawn) positioned herself more powerfully in the two friendship and romance discourses and hence engaged further in mathematical practices. We note that romance and friendship discourses may share commonalities with common understandings of love but are not exactly the same.

The literature on affect in mathematics education also connects to love. For example, Debellis and Goldin (2006) argued that “intimate mathematical *experiences* include emotional feelings of warmth, excitement, amusement, affection, sexuality, time suspension, deep satisfaction, ‘being special’, love, or aesthetic appreciation accompanying understanding” (pp. 137-138). They also pointed to the fact that individuals’ accounts of loved ones—for example, “a parent being proud of me”—point to experiences that are “more than merely enjoyable or otherwise positive; they *build a bond* between the personal knowledge constructed and the mathematical content” (p. 138). In other words, these researchers argued that emotional feelings are important for the students’ loving (or unloving) relationships with mathematics and/or mathematics education and that these emotions may originate in love, care and appreciations for significant others. Hackenberg (2010) presented another approach to affect. She defined “mathematical caring relation (MCR) as a quality of interaction between a student and a teacher that conjoins affective and cognitive realms in the process of aiming for mathematical learning” (p. 237). She elaborated:

Mathematics teachers act as carers in general, but they start to act as mathematical carers when they hold their work of orchestrating mathematical learning for their students together with an orientation to monitor and respond to fluctuations in positive energy available to the self. (p. 237)

We also found the idea of *pedagogical love* (van Manen, 1991). Curriculum studies researcher Hatt (2005) reminds us that “pedagogical love cannot be perceived, nor received, in a curriculum environment in which feeling for the Other is absent or marginalized” (p. 671). In mathematics education research, pedagogical love is exemplified by the Finish teacher Sirpa, in an account written by Kaasila (2007), wherein Sirpa’s thoughts about her teaching and relationships to her students is captured in her use of the utterance of pedagogical love: “I want to be a teacher who really cares how she acts. Pedagogical love—the word feels suitably descriptive. It includes caring for oneself as a teacher and as a human being” (p. 380).

This reminds us of the idea of *fostering by love* (relating to the idea of “tough love”) in the ways the teacher Ms. Bradley acted in her classrooms, as described by Bonner (2014, p. 395). This may seem antithetical to caring love because it ignores the other’s perceived needs from a paternalistic standpoint of knowing those needs better:

Ms. Bradley's classroom was highly structured and disciplined, focusing on high expectations and success through "tough love." When a student did not have his or her homework, for example, Ms. Bradley would take the student in the hallway to call his or her parent or guardian. Furthermore, if a student was not participating in the group chants or problem-solving activities, Ms. Bradley would "call him [or her] out and take him [or her] to church," meaning she would stop the lesson and "preach" about the decisions students were making and the importance of academic success.

Long (2008) suggested that student mistakes in mathematics provide another context for examining care. She argued that teachers might respond to students' or pupils' mistakes in two different ways, with a tension between either caring for the student or caring for the idea of mathematics. In her words: "Specifically, I think there are analogous tensions in caring for people and caring for mathematical ideas" (Long, 2011, p. 2). We see this tension as analogous and at times overlapping with the tension identified by Bartell (2013)—"a tension in negotiating mathematical and social justice goals" (p. 140).

We reflect that both MCR and pedagogical love seem to suggest that the acts of love are separate from the disciplinary work—"I teach mathematics and I also show love to my students." Noddings' (2010) elaboration on caring helps us see the possibility for furthering mathematical goals by caring for others in dialogue: "To learn what the cared-for is going through, or what the carer is aiming for, and then to work cooperatively on meeting the needs" (p. 147) may apply to mathematical and extra-mathematical goals. We argue that people can love (or show love) through the way they do (or communicate) their mathematics. And by contrast, one can bully using mathematics. Interestingly, our literature review did not identify any literature that focused on how mathematical language communicates love. In this article, we begin to address this omission.

## **Bullying**

Intimidation and humiliation are typically present in bullying situations, especially among school children, but also among adults or between adults and children. Bullying may include threat, physical assault, or what we noticed in our data—verbal harassment and sarcasm. Like love, the ideas of bullying and intimidation have various nuances. We follow the definition of bullying by Juvonen and Graham (2014) who described bullying in contexts involving inequalities, such as when a "physically stronger or socially more prominent person (ab)uses her/his power to threaten, demean, or belittle another" (p. 161). The authors emphasized that this specific power imbalance distinguishes bullying from conflict. Thus, bullying can be seen as an antonym for loving. Instead of being responsive to the needs of others in a caring way, bullies either respond to their needs with dismissiveness or ignore their needs in contexts where there would be a reasonable expectation for caring. Studies of youth experiences indicate that approximately 20–25% of youths are directly involved in bullying as perpetrators, victims, or both (Juvonen & Graham, 2014; Nansel et al., 2001).

A large body of research shows that bullying significantly impacts students' achievement in general (for overviews, see Hawker & Boulton, 2000; Nakamoto & Schwartz, 2010). Fewer studies

specifically focus on the lower achievement in mathematics that is particular among victims of bullying in the elementary school years, and fewer studies yet consider this in high school contexts (Juvonen & Graham, 2014). Yetter's (2011) thesis showed that students' achievement in mathematics in particular increased when teachers regularly had anti-bullying classroom meetings specifically designed to "build relationships within the classroom where students feel welcomed and safe to learn" (p. 2).

Bullying has been described as one important reason for not pursuing mathematics. Sullivan, Tobias and McDonough (2016) described how students might avoid trying to do the required mathematics even when they are good at it. As an example of this, we also found Lutovac and Kaasila's (2014) narratives of Reija, a pre-service teacher: "Reija's self-confidence and participation in math classes worsened in secondary school because her classmates bullied her: 'I tried to be as invisible as I could'" (p. 136). Reija did not pursue her mathematics education.

Some mathematics educators have described mathematics itself as bullying. In the foreword to a book by Davis (1996), Pirie implied that the positioning of mathematics with privilege in decision-making is a form of bullying:

Let us construe mathematics not as a human endeavor, but as itself a living being. . . . As with any persuasive orator, it speaks eloquently, but from a personal perspective that we can decide to ignore. Its world view does not have to be ours. It is saying, "here is a way of being, and in this being lies a (but not the) potential for growth and change." How might such a re-envisioning of mathematics affect the classroom? How much less frightening might it seem to children? What if they were encouraged to resist its bullying and to see its problems as living possibilities and not as mandated chores? (p. xiv)

Gutiérrez (2017) went further, fingering abstraction itself as a source of trauma:

So many people are walking around in society who have experienced trauma, microaggressions from participating in math classrooms where the idea of being a successful person, being an intelligent person, is removing oneself from the context, not involving emotions, not involving the body, and being judged by whether one can reason abstractly. (p. 18)

As with loving, we problematize the idea of mathematics doing the bullying or being aggressive, and instead we consider how it might bully through conversational interactions and texts in the classroom.

Finally, we point to research that describes tense relationships among students when doing group work and/or mathematical investigations. Kurth, Anderson and Palincsar, (2002) showed that students might fail to learn during group work due to the fact that they need to achieve intersubjectivity in relation to both knowledge assimilation and obligation within the group. Specifically, the students' needs and obligations in the group were varying and sometimes even in conflict. These researchers concluded that "even when [teachers] cannot monitor directly what is happening in collaborative groups, [they] still have considerable influence over how students construct the floor, both in terms of who holds the floor and what topics the students consider 'on

task” (p. 310). Andersson (2011) came to similar conclusions (see Andersson & Valero, 2015). In a critical mathematics education context, three students were working together in a project and experiencing some tension—probably not to the point of bullying though. One of them commented on the classroom blog when the project was done: “This was really meaningful and it was good to take personal responsibility for planning and for our own labor. But this is new; we have to practice this way of working” (Andersson & Valero, 2015, p. 212). The problem for this group was not the mathematical task at hand; it was how to collaborate when exploring mathematics.

Summing up, the research on bullying indicates that bullying occurs in interactions—hence through the use of language. This is specifically important to consider for teachers when students do collaborative work in mathematics as it not only impacts students’ wellbeing but also their achievement—even if they love the subject.

### Solitude

We see solitude as another antonym for loving. We describe love as relational. By contrast, in solitude one removes oneself from relationship, usually temporarily. We are defining solitude as a state of seclusion or isolation, in other words a lack of contact with people. It is not the same as silence. Xu and Clarke’s (2019) challenge to Western assumptions in scholarship of classroom discourse highlight the importance of silence in some cultures and its connections to wisdom and thus engagement. Even Western traditions recognize silence as part of interaction: for example, Loy’s (2010) quote, found in Latremouille’s (2018) poetic work on silence, schooling and bodies, where silence is interrupting talk—“Not-speaking may be part of a story, even as the pauses in music are part of the music. Sound need not break the silence. It can be an expression of the silence” (p. 54).

While there is a paucity of research relating to solitude in mathematics education, we find that the research there is points to the prevalence and significance of solitude in mathematics classroom experiences. A body of research has shown that students perceive mathematics as an isolated subject where few opportunities are offered to work or discuss with friends (Andersson, Valero, & Meaney, 2015; Boaler, 2015; Nardi & Steward, 2003). We see elements of solitude in Skovsmose’s (2001) description of silent counting or individual task work and textbook work—generally work within the “exercise paradigm” usually described as “traditional mathematics teaching”.

Previous research from psychological perspectives and constructivist theories have emphasized the specific importance of solo work in mathematics for individuals’ learning (Wilson & Peterson, 2006; Wood, Cobb, & Yackel, 1991). Pimm’s (1987) psychoanalytical reflection on mathematics places silence as central to a desire for permanence, invariance and infinity, which ultimately “involves the removal of active human agents” (p. 37). While our field’s awareness of social aspects of learning has grown (e.g., Lerman, 2000), individuals’ work in solitude, or maybe a wished-for peaceful silence, still seems to be commonplace in mathematics classrooms.

We are reminded of Andrew Wiles, the mathematician who worked on Fermat’s last theorem for years in solitude. It was not until he presented his work publicly that his solution was developed to become an accepted solution (Singh, 1997). We believe that both solo work and collaborative

work with others each have their advantages at different points in time. However, the nature of solitude—whether it is deliberately set up in mathematics classrooms or not—surely impacts student experiences of mathematics. Our illustrative episodes show that solitude may even be present when students are collaborating.

### **Our Analytical Frame**

Aspects of power and authority are central to our focus on loving, bullying and solitude. Perhaps they are most obvious in bullying situations, but we would hope that loving is also possible in contexts that have power and authority at work. And we see solitude as another response to experiences of power and authority. Mathematics education can be described as a series of created and re-created practices within social, cultural and political contexts. Power is distributed among the actors in such contexts. In line with Gutiérrez (2013) and Valero (2004), we understand power as situational, relational and in constant transformation. Power obviously works between these contexts as macro-level processes; however, power also works at the micro-level in the immediate situational contexts between participants. Macro-level practices give meaning to micro-level actions, offering participants subject positions. However, the participants' micro-level actions also give meaning to the macro-level practices and thus participants position themselves in ways that are reflexive, relational and contextual in relation to the discipline and to other individuals in their learning contexts (Herbel-Eisenmann et al., 2015). Micro-level communication acts are the focus in this article.

In this article we follow Foucault's (1972) sense of the term *discourses*—"practices that systematically form the objects of which they speak" (p. 52). However, a discourse is also about "negotiating and maintaining relationships among its participants" (Morgan, 2012, p. 181) and hence establishing relations and positionings. This way of thinking about discourses aligns with *figured worlds*, as identified in the work of Esmonde and Langer-Osuna (2013), which we referenced earlier, and with *storylines*, as described in positioning theory (Wagner & Herbel-Eisenmann, 2009).

Our analysis in this article will focus on spoken communication in group work. We are most interested in relationships of love, bullying and solitude, but we have come to realize that these do not exist in single communication acts. They may be read into a particular communication act on the basis of a pattern of interaction, but we cannot see a single communication act as an act of love, bullying or solitude, nor can we identify these things from one day's group work. We recognize that communication acts may include gesture and other paralinguistic features, but we focus at this time on the words spoken in dialogue.

Our analytical frame makes distinctions that can be attributed to individual communication acts, but it does not identify love or bullying. Indeed, when we presented our initial work on this framing, our audiences were resistant to the extremes of love and bullying; they thought there should be something in between. Our conversations with them, and among ourselves in relation to our data, motivate us to agree with their concerns. We cannot read love or bullying into one act. However, we can identify whether a communication act responds to or dismisses the preceding communications. Because language by nature makes distinctions, we argue there is no "in between"

on this distinction. Patterns of response and dismissiveness, however, form the possibility of interpreting love, bullying or some kind of in-between state that may be an emotion-filled ambiguity or a less emotive state. The communication acts are accessible to us, while the intentions and interpreted emotions are not as identifiable.

Our analytical frame comes from a recursive process starting with the data that prompted our attention to love and bullying, connecting that to literature and conceptual frames we knew, trying to use these frames for analysis, finding difficulties, modifying the frames, trying again, etcetera. We describe the frame here, before illustrating its application, in keeping with the traditional structure of academic articles, but we emphasize that the development of the frame was intertwined with the analysis. Figure 1 gives an overview of the frame we developed.

To elaborate on the development of this frame, we first point to the conceptual frame for identifying authority structures refined by Wagner and Herbel-Eisenmann (2014). It helps us see how students make space for each other in their interaction, which relates to the idea of being responsive to each other or not. This distinction between being open and closed often appears in literature on interaction. Appraisal linguistics uses the terminology of *heteroglossia* to refer to communication that opens space up for diverse perspectives and *monoglossia* to refer to communication that closes space off (Martin & White, 2005). When people close off space for others to exercise autonomy, they can do so in a number of ways, which the frame identifies. We initially thought of making space for others as an expression of love, but our analysis of data, as illustrated below, and our reflection on experience, clarify that a bully may also invite someone to make decisions—perhaps as a way of trapping the person into failure. Likewise, a loving person may protect someone from decisions beyond their capability in a particular context, as described in the literature on pedagogical love and fostering love.

Using Wagner and Herbel-Eisenmann's (2014) conceptual frame, we identify three different ways in which autonomy is closed off. The indicators of these authority structures in this frame enable analysis. In the *personal authority* structure "someone is following the wishes of another for no explicitly given reason" (p. 875), in other words, autonomy is closed off by the status of someone in the interaction. Linguistic clues may include the presence of the pronouns *I* and *you* in the same sentence, exclusive imperatives (Rotman, 1988), closed questions (Martin & White, 2005) and choral response. In the *discourse as authority* structure, space is closed off by an idea or discourse that is larger than the relationships in the interaction. To identify this authority structure, one looks for "evidence that certain actions must be done where no person/people are identified as demanding this" (Wagner & Herbel-Eisenmann, 2014, p. 875). The strongest linguistic clue is the presence of modal verbs that suggest necessity—e.g., *have to*, *need to*, *must*. A more subtle form of this structure was called *discursive inevitability* by (Wagner & Herbel-Eisenmann, 2014), but we found that this description does not describe all the interaction that fits the category, so we are calling it *spoken as shared*. For this authority structure, people speak as though everyone agrees about the way things are and the way things will happen without giving reasons for how they know. The simple verb *is* and the modal verb *going to* are strong indicators of this structure, as in "this is a triangle" or

“this is going to simplify”. Also, counting outside sentence structure indicates that no one would disagree with the count (Andersson & Wagner, 2018).

Wagner and Herbel-Eisenmann (2014) identified only one authority structure describing an open space for autonomy—*personal latitude*. We have divided this into three different ways of opening space or acknowledging that others have autonomy. We wondered if it is necessary to distinguish among these, as it is most significant that space is opened up, and it may not be significant *how* one opens space. By contrast, we think it is significant what authorities are used to close space. Nevertheless, our three categories help us identify moves people make to open space. We identified *explicit invitations*, in which someone asks for another’s point of view. Linguistic indicators include open questions (Martin & White, 2005) and inclusive imperatives (Rotman, 1988). In another authority structure, someone contributes an idea but identifies that it is a possibility or that someone else might think differently. We call this *spoken as possibility*. This is typically done with hedges (Rowland, 2000). Finally, explicit reference to choices, which we call *identifying decision*, demonstrates awareness of potential alternative possibilities, typically indicated as recognition that a different decision could have been made—for example, “I was going to” or “he could have”.

While there are various ways of speaking that open or close autonomy for others (which we represent on the vertical axis of our diagram in Figure 1), there are other dynamics at work, which also relate to love, bullying and solitude. Following our reading and reflection on literature describing love and bullying, we identified responsiveness as central to love. We struggled to clarify the complement to the set of approaches that are responsive. Dismissiveness seems like an opposite of responsiveness, but our data and reflection helped us see that besides overt ignoring of others, any move to be dismissive is really a move to foreground something else above the needs or interests of another person—that is, responsiveness to something or someone else deemed more important. Thus, the key question is this: To what/whom is the speaker responsive? Nevertheless, we will use the distinction of responsive versus dismissive and we focus on being responsive or dismissive to the people in the interaction. If someone is responsive to a demand from outside the interaction and thus dismisses the voice of someone in the interaction, we classify the communication as dismissive. This distinction appears in the horizontal axis of Figure 1.

The analytical frame we have developed does not represent solitude because it means, by definition, a lack of interaction. Thus, we are attentive to the discourse that invokes solitude. Even so, we realize that many instances of solitude are prompted by interactions that precede any particular interaction that we analyze.

		<i>looking back</i>	
		Responsive (R)	Dismissive (D)
<i>looking forward</i>			
Opening space (heteroglossia) (inviting)	Spoken as possibility (SP)	SP-R	SP-D
	Invitation (I)	I-R	I-D
	Identifying decision (ID)	ID-R	ID-D
Closing space (monoglossia) (restricting)	Personal authority (PA)	PA-R	PA-D
	Discourse as authority (DA)	DA-R	DA-D
	Spoken as shared (SS)	SS-R	SS-D

Figure 1. Analytical Frame

### Classroom and School Context

The data that drew our attention to student language repertoires for loving, bullying and solitude in mathematical contexts comes from a Canadian high school with students from Grades 9-12. Not surprisingly, the school promoted loving relationships, not bullying. We saw posters and pictures on hallway and classroom walls with proverbs and metaphors clarifying expectations for the school’s people to be kind, to listen to others, and to act in responsible manners. It was obvious to us that the presence of these institutional discourses (Foucault, 1972; Gee, 2012) with the keywords of *respect*, *responsibility* and *commitment* may be interpreted as aspects of love. The presence of these reminders also suggest that school personnel felt that students and perhaps others in the school required reminders to love in this way.

We also noticed a prominent display identifying the “Mathematics students of the month” from each grade. Values such as respect, responsibility and sharing were not honoured in this way, but we do not know enough about the school to say how students embodying these values were otherwise honoured. Nevertheless, the walls bore requests and encouragements for children to relate in loving ways to each other, and honoured children who achieved well in mathematics. Given that victims of bullying generally perform poorly in mathematics (Hawker & Boulton, 2000; Lutovac & Kaasila, 2014; Nakamoto & Schwartz, 2010; Sullivan et al., 2006), the “Mathematics student of the month” awards may even further marginalize victims by honouring the people who succeed in a system that makes success difficult for victims of bullying. This could be a case of reproduction of power and inequalities (Foucault, 1972, 2002).

In this school, we worked with mathematics teachers to engage students in a mathematical investigation task. The teachers showed their students a red painted solid cube (see Figure 2) and presented this problem:

A cube was painted red, and then cut into smaller cubes,  $3 \times 3 \times 3$ :  
How many of the small cubes have no red faces?  
How many have 1 red face? 2 red faces? 3 red faces? 4 red faces? 5? 6?  
How about a cube cut into  $4 \times 4 \times 4$ ? Or  $5 \times 5 \times 5$ ? Or  $10 \times 10 \times 10$ ? Or  $n \times n \times n$ ?  
What if it is cut into  $3 \times 5 \times 8$ ? Or  $n \times m \times p$ ?

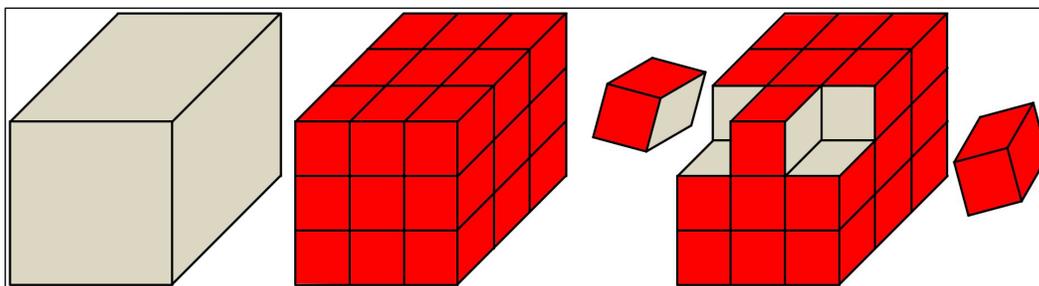


Figure 2. Red Painted Cube. Image by D. Wagner.

The teachers arranged the students in groups of three and four. The students who consented to video- and audio-recording worked in groups together. The other students also did the task but were not recorded. As researchers and teachers, we agreed to interfere with the students as little as possible—we would not go beyond answering clarifying questions. The students were given sticky note pads, a unique color for each group. They were asked to write their significant findings on these notepapers and post them on the board. The students spent about 50 minutes on the task.

In the illustrative transcripts we use gender neutral pseudonyms because we did not have sufficient evidence that the responsiveness, dismissiveness and withdrawal was impacted by gender. We understand that identity characteristics of students are impactful, but it would take much more data or clearly articulated speech acts to warrant claims about identities in relation to our analysis. We agree with Noddings (2010) that caring is gendered. We add that it is also culturally situated, and that bullying and solitude are also gendered and culturally situated. Again, our data excerpts are illustrative of the way students use language, but they are too brief to make claims about the roles of culture and socio-economic status in these interactions.

All the excerpts used below come from the same class, to emphasize that loving, bullying and solitude are likely present in every class (which is not the same as saying they all belong in the classroom): We do not blame the teacher for their presence. For each of love, bullying and solitude, we have selected example excerpts that suggest the relationships clearly and other excerpts that demonstrate more complexity. We think it is important to choose episodes that push at the boundaries of analytical frames.

## Interactions That Suggest Love

This first excerpt is chosen because it seemed to represent a relatively clear indication of love among the four students in the group—supportive response to each other and caring. Participant names are pseudonyms. We include in the right-hand column of the transcripts our coding using the acronyms from Figure 1.

A15	Adrian	Do you have to write the question down?	DA/ID-R
A16	Blake	I don't know.	SP-R
A17	Kelsey	I wouldn't, I'd say that just one block has no red.	ID-R
A18	Adrian	Um, how many have one red face?	DA -D
A19	Kelsey	So, it would be like . . . wait . . .	SS-R
A20	Dana	[ <i>points</i> ] This one . . .	SS-R
A21	Kelsey	This one. [ <i>spoken in rapid succession</i> ]	SS-R
A22	Blake	Two. [ <i>. . . rapid succession</i> ]	SS-R
A23	Adrian	Two, three. [ <i>. . . rapid succession</i> ]	SS-R
A24	Kelsey	Four. [ <i>. . . rapid succession</i> ]	SS-R
A25	Adrian	Two, four, six. [ <i>. . . rapid succession</i> ]	SS-R
A26	Kelsey	Six. [ <i>. . . rapid succession</i> ]	SS-R
A27	Adrian	Yeah. [ <i>. . . rapid succession</i> ]	SS-R
A28	Kelsey	It would be six. [ <i>. . . rapid succession</i> ]	SS-R
A29	Kelsey	because there was . . . Is it only six?	SS-R SP-R
A30	Blake	We'll figure it out at the end. We'll see if we've accounted for twenty-seven blocks.	SS-R
A31	Kelsey	Okay.	ID-R
A32	Blake	So, two red faces . . .	DA-D/R
A33	Adrian	two red faces, okay, so . . .	SS-R

We first consider how the group participants opened and closed space for each other's views. Adrian [turn A15] appealed to the authority of the discourse (DA) asking if they "have to" write. Perhaps this is what they were accustomed to from their usual mathematics classes. However, Adrian also posed this as a question and thus identified the group's latitude to make a decision (ID), which opens a space for her partners' ideas. Blake's matter of fact response "I don't know" [A16] indicated awareness of possibilities (SP). And Col kept the space open [A17] using the modal verb *wouldn't* to recognize choice (ID). Significantly, this open space, already communicated by three of the students in succession, led into Kelsey's opening of mathematical possibility, with a conjecture—"I [would] say . . ." Adrian continued the open approach to the mathematical exploration by inviting a decision (ID)

[A18]. This led into a rapid series of observations, which we coded as spoken-as-shared (SS) because the students were trying to answer their question. While this structure is generally a closed approach, its appearance in response to a group's question may also be interpreted as honouring the opening up question. Eventually [A29], Kelsey again asked a question identifying possibility (SP), and Blake responded with an expression of confidence in the group, immediately followed by a means for checking their work. Kelsey's response [A31] indicates the expectation that agreement was necessary (ID), and Blake returned the group back to following the instructions and personal authority (PA) of the teacher.

In this environment of openness to each other, we coded the students as responsive to each other (R), except when students brought the groups' attention back to a question on the instruction sheet [A18, A32]. However, even these moves that apparently ignored (or dismissed) what came before (D), may in fact be tacit recognitions that the group was ready to move on, in which case one could claim that even these moves to follow the instructions demonstrated responsiveness to the group's progression.

The responsiveness is an indication of sensitivity to each other, and even the moments of returned attention to the task given by the teacher may be deemed as sensitivity to the group's desire to move forward mathematically. These can be interpreted as indicators of care for the other, or what we call love. Other indicators of this love include the use of the pronoun *we*, indicating a sense of solidarity, and an intense series of observations [A21-28], in which the four students were huddled close together, with their hands in close proximity, pointing to parts of the cube model. The mathematical work was also productive within the open and caring interaction in this group.

The next transcript excerpt focuses on the work of a group that was challenged by the mathematical task and which at times diverted attention to other aspects of the students' lives, namely friendship and romance. The group also joked about the task at times. Here, we meet them about half way through their work on the task, when they were addressing the first part of the task. Storm took active steps to include everybody in the interaction. We chose this transcript because it exemplifies two students interacting along a teacher-student storyline, and because it is less clearly an instance of loving care for each other.

B108	Storm	They'll fit one way or another. Shoot, I . . . [ <i>Gets up</i> ] There, that'll help. [ <i>attaching the final block in a 4 x 4 x 4 cube</i> ]. So, we already know that four would have no red faces. Two red faces. One, two, three, four, five, six, seven, eight, . . . nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen. . . . Oh my god, there's twenty-four. [ <i>blocks fall off the structure</i> ] Okay, Parker, . . . I don't want to lose you. How many do you think would have three red faces? We already figured out the ones with the two red faces.	SS-D ID-D SS-D  ID/PA-R I-D SS-R
B109	Parker	How many would have how many red faces?	I-R
B110	Storm	Three red faces. Like painted red faces.	SS-R
B111	Parker	Yeah, it's each corner isn't it? So, so that's one, that's two . . .	SP-R SS-R/D
B112	Storm	Oh, so it would be the same.	SS-R
B113	Parker	. . . five, six, seven, eight. Yeah.	SS-R
B114	Storm	It would stay the same.	SS-R

After being flummoxed by a desire to build a 4 x 4 x 4 cube with the blocks and eventual success on this, Storm [B108] set the tone of this part of the conversation by stating observations as if they were shared (SS). Storm was counting blocks outside of sentence structure and making claims about the group's knowledge. Apparently positioning self as teacher, Storm worked through the task step by step and used the inclusive pronoun *we* in a leading way with little indication that this reflected communal work (see Pimm, 1987). Storm's aside spoken to Parker (still turn B108)—"I don't want to lose you"—is also very teacher-like. It is recognition of a social decision (ID) to include everyone and also an indication of a sense of responsibility or personal authority (PA). When Storm addressed Parker with an invitation (I) to say how many cubes would have three red faces, it is unclear, even from the gestures, whether the others felt invited to answer. Nevertheless, this move opened the space for Parker to ask a clarifying question (I), and to make a mathematical conjecture (turn B111). The conversation continued with Parker and Storm making observations, but apparently building off each other.

The beginning of this exchange had Storm distracted with manipulating the physical cubes and thus dismissive (D) of the group. Once Storm got going on observations, Storm continued to ignore the group mates (D) until the exchange with Parker, in which it appeared that they were responsive (R) to each other. It is unclear to us whether Storm was asking an authentic question of Parker and informed by the reply, or if Storm was checking to see if Parker understood her process and confirming the reply. Nevertheless, the teacher-student positioning seems to suggest Storm's care for Parker to understand the mathematics. This is not necessarily the same as caring for Parker. It may instead be an expression of care for the mathematical success of the group (similarly, we might

distinguish between a teacher’s care for students and care about having good results connected to his/her name). Because the pattern of dialogue between Storm and Parker so well resembles a teacher-student interaction, we could say that they are more responsive to the teacher-student storyline than they are to each other. If this is the case, we might think of them being dismissive of each other and more responsive to the discourse of school mathematics.

### **Interactions That Suggest Bullying**

Next, we consider interactions that seemed to represent some level of bullying. The three transcripts feature a group of four: Andy, Dana, Jesse and Kai. The first transcript is from very early in their conversation. Before addressing the mathematical task, they discussed the researchers’ voice recording equipment. This gives us an introduction to the nature of the relationship. As with the transcripts representing some level of love and care, we analyze with our framework to consider how responsiveness and dismissiveness may develop antagonism.

C10	Andy	Yeah, we also found out what these were, so we are not gonna say anything dumb.	SS/PA-R
C11	Dana	So we’re not gonna say Kai’s name.	SS/PA-R/D
C12	Kai	You guys are jerks. You guys are mean. A cube was painted red and cut into smaller cubes, three by three by three. Okay guys, okay. We gotta work. How many small cubes have no faces?	SS-R/D DA-D I-D

Andy [C10] took the lead by making statements as if they were shared (SS) and thus expecting that the others would follow this personal authority (PA). Andy didn’t want the group to say anything “dumb” on the voice recording. This double-move (SS and PA) closed the communication space to a certain kind of interaction. Dana’s response seems to respond (R) to Andy but put a spin on it to exclude Kai and thus the move was dismissive (D) of Kai’s worth and dismissive of Andy’s suggested positioning. Dana’s voice was similar to Andy’s with a statement of fact, vying for personal authority. Next, Kai challenged Dana’s insensitivity, which is a response (R) but also dismissive of his meanness (D). Kai then sought to turn attention to the discipline as authority, calling on the mathematical task to guide their interaction [C12, line 2]—“We gotta work”—and then inviting (I) participation along that storyline—“How many small cubes . . .?” This was dismissive of Dana’s positioning.

We do not see indications of love in this interaction. It appears to be a case of Dana being mean to Kai, who resisted. We know from experience in schools that such interaction among students is often positioned as friendly banter, and we wonder about the impact of such complicity from adults responsible for school children.

We pick up the conversation a little further along when the group is engaged with the mathematical task.

C35	Andy	For the eight corners, you're gonna have three sides facing. One, two, three. So, eight times three?	SS-D I-R
C36	Kai	Fifty-four.	SS-R
C37	Jesse	[Laughs] No! You're a dummy.	SS-R
C38	Dana	[Interrupting] Twenty-four.	SS-R
C39	Andy	So, red faces. We'd have to have eight because there's eight corners, three sides exposing. There. One, two, three, . . . eight corners. One, two, three, four, five, six, seven . . . For that you just write beside it, "eight."	DA-D SS-D PA-D

In turn C35 Andy was responsive to the task set by the teacher and to Kai's move to turn attention to that task. Andy was thus responsive to the personal authority (PA) of one or the other. Andy spoke as if the others would share the conclusion (SS)—"you're gonna have", which closed the conversation space, but then Andy opened it up by inflecting "eight times three" as a question, inviting response (I). Kai was responsive and answered with no sentence, just a number (SS) [C36]. Unfortunately, this answer was incorrect. Jesse responded with ridicule, and Dana responded with the correct answer. Andy [C39] ignored the wrong answer and the responses to that wrong answer, turning attention again to the discourse as authority—"we'd have to have . . ."—and continued speaking as if there were no options (SS). Andy finally instructed the others what to do, claiming the personal authority to do so (PA).

This interaction is a clear instance of responsiveness not being loving—as Jesse responds with ridicule to Kai's mistake. We could instead say this is dismissive of Kai's worth. Following this, Andy dismissed the group mates' interaction, ignoring it really. This move may have been an act of love for Kai—forgiveness of Kai's mistake, and rejection of Jesse's ridicule. This episode demonstrates how dismissiveness and responsiveness do not neatly map onto bullying and love. It also exemplifies the way mathematics can be a refuge for people who are bullied, which is a reality both of us have seen in our school teaching experiences.

The last transcript from this group is further into their conversation. The group has not moved forward very much mathematically. Andy was still the one taking leadership with personal authority but was beginning to realize that his mathematical ideas might not work.

C78	Andy	[ <i>Takes the cube</i> ] because in the middle here, it's covered because one, two, one, two . . .	SS/DA-R
C79	Jesse	Oh, so only the ones in the middle.	SS-R
C80	Andy	That's it, we have trouble.	SS/SP-R
C81	Jesse	I am an idiot.	SS-D
C82	Andy	We all are.	SS-R
C83	Dana	[ <i>speaking to Jesse</i> ] Compared to Kai, you're a genius.	SS-D
C84	Kai	[ <i>Not speaking, standing up dramatically</i> ]	
C85	Jesse	I'm really not guys. I am bad at math.	SS-R

In turn C78, Andy took the cubes suggesting ownership and gave an explanation as if it was the only possible answer (SS). Because this communication included explanation Andy was harkening to the mathematics discourse as authority as well (DA). Andy continued to close the space for the others' contributions—even if it seemed as if Jesse had shared some possible ideas earlier [C79-80]. Jesse was responsive to Andy's explanation and continued with the sense that there is only one correct path (SS). Andy responded to this and referred back to a statement by Dana, which showed Andy's realization that something was amiss. The form of this communication—"we have trouble"—suggests that everyone would agree, but it also acknowledged that there might be more ways of seeing the situation (SP). Jesse claiming to be an "idiot" was a self-judgment, and thus Jesse ignored the others. After this, the positioning within the group went awry, with more ridicule of Kai [C83]. The space for Kai to speak or otherwise contribute was closed, almost locked.

While the group members expressed their feelings of stupidity, there was again especially strong ridicule of Kai [C83]. We see that the mathematical task along with the traditions of mathematics classroom storylines (see Gutiérrez, 2017), positioned the group in this way—feeling stupid, ranking their levels of stupidity, and thus positioning one person especially poorly. This reminds us of Solomon's (2009) elaboration of the discourse that says you are dumb if you do not know certain mathematics. We ask if there was any love here. Perhaps there was a sense of care among Andy, Dana and Jesse as they formed solidarity around the exclusion of Kai. Perhaps Andy's recognition that the whole group was on common ground as they could not come to terms with the problem [C82] was a way of expressing camaraderie in recognition of the feelings of stupidity. Nevertheless, the one-right-answer tradition of school mathematics seems to have laid the ground for feelings of ineptitude and for ranking, which positions one or some as the lowest of the low.

### **Interactions That Suggest Solitude**

To complete the love-bullying-solitude triad we now at last turn to solitude. Earlier we described solitude as a state of seclusion or isolation, in other words a lack of contact with people. However, this does not necessarily mean doing mathematics alone as if being on a desert island. As shown below, one can experience solitude even in group work. As we indicated above, solitude is

odd to identify because it appears as an absence. However, communication acts may well motivate someone’s reclusion.

In the first excerpt, we focus on the interaction among Dallas, Jordan, Quinn and Kelsey. Quinn was silent for the entire hour except for three short utterances (two of them were single-word utterances). However, Quinn seemed to be listening and watching the writing and manipulative use of the others. Leading into this excerpt, Quinn was silent as usual, and apparently watching intently.

D168	Jordan	Let me see this. [ <i>Grabbing the cube</i> ]	ID-R
D169	Kelsey	Yeah, there’s nine inside.	SS-R
D170	Jordan	For four or for five?	I-R
D171	Kelsey	So, three by three equals . . . [ <i>starting to draw</i> ]	SS-D
D172	Jordan	[ <i>Talking to Quinn</i> ] Are you getting any of this?	I-R
D173	Quinn	I’m confused.	SS-R
D174	Jordan	They’re so far ahead [ <i>gesturing toward another group</i> ]	SS-D

Before and after the above excerpt, Jordan, Kelsey and Dallas had been very active and were working with the task, moving their mathematics forward in a friendly manner. They had been laughing, shifting among different discourses—for example, engaging in mischief such as putting pencils between their noses and lips and laughing towards the camera: “That’s so hilarious! Show the camera!” [D273, Jordan].

Our analysis showed that overall the group’s work as responsive to each other with this pattern suggesting love. And we wondered whether to include Quinn as part of this circle of love. In this excerpt, Jordan [D172] explicitly opened space for Quinn to speak by asking if Quinn followed. Quinn’s response [D173] “I’m confused” is difficult to interpret. It may have been a plea for help. The vagueness of this self-assessment made it difficult for the others to respond, and thus Quinn’s self-assessment may have been a masterful way of turning the attention of the other three group members away again. Thus, Jordan’s response [D174] to Quinn—in which Jordan switched attention back to the progress of the group and away from Quinn—may have been dismissive of Quinn’s plea, or may have been a gentle acquiescence to Quinn’s wish for privacy/solitude. We concluded that the only way to read this situation (in the video or the transcripts) would be in a much larger context of these students’ interaction. Nevertheless, we are left wondering what might have motivated Quinn’s silence. It may have been mathematics class discourse, home discourse, or any number of experiences.

Thus, we turn our attention to a situation in which it appears that people push another into solitude. (We do not wish to imply that Jordan’s situation would have been analogous.) We return to Kai, Andy, Dana and Jesse.

C370	Kai	Guys, if it goes up one number . . . Guys? Andy? Andy? I am trying to help. Why can't I help?	SS-D I-R SS-R I-R
C371	Andy	Shhh. It's eight.	SS-D
C372	Dana	We just need one more cube.	SS-D

As reflected in our earlier analysis, Kai was called silly, dumb and stupid, and was otherwise ridiculed before the excerpt above. Kai persisted to try to communicate though substantive contributions were regularly dismissed. Here [C370] Kai switched back and forth between the conventional, closed mathematical talk of making observations that are spoken as shared (SS) and more open explicit invitations (perhaps “pleas” would be a better word) for the others to listen. Finally, Andy shushed Kai [C371]—“Shhh”—and the others continued in the interaction ignoring (dismissing) Kai’s contributions. Kai was there in the group, and even making progress mathematically, but nevertheless working in solitude. Perhaps this was a harsher solitude than desert island isolation because the potential interlocutors were at hand but unresponsive.

### Discussion

Our analytical frame and its use illustrated in some brief interactions, helps us understand how mathematical discourse is used to love, bully or promote solitude, all important dynamics in mathematics classrooms. This study, which was conducted with high school students working in groups on mathematics problems, addresses students’ conversational interactions in a way that could lead to a better understanding of social groups in high school mathematics contexts and how to make those groups function better. It could also lead to a better understanding about how to engage students in mathematical conversations, how to motivate them, and how to foster their self-esteem.

To reflect on our analytical frame and its application to the example episodes we begin with thoughts about solitude. We ask ourselves what the role of mathematics classroom discourse may be in the above situations of solitude. Is there something about the discourse that left Quinn disinterested, or perhaps quietly engaged? Of course, it would have to be something about the way the discourse was expressed for Quinn, but it would be connected to the larger discourse, nevertheless. And, as we asked in our discussion about bullying, how did mathematics classroom discourses (in this classroom context and others) lay the ground for the group’s ridicule of Kai?

This question about the role of mathematics classroom discourses and practices in initiating and sustaining solitude or interactions that are loving or bullying is beyond the scope of our analytical frame. However, it is an important question that arises from our analysis and begs further attention. In particular, we suggest that further research analyses focus on an individual, or a few individuals, to gather a wider range of data that documents their experiences of interactions in mathematical learning contexts. This kind of deeper analysis into one person or group could also enable consideration of the role of student identities, particularly gender, culture and socio-

economic status. Nevertheless, we can say from our rich data, exemplified with the very short excerpts analyzed above, that the way communication happens in mathematics classrooms can open or close space for the other, and that this impacts students' experiences of love, bullying and reclusion.

We highlight the importance of attention to their responsiveness to each other (which, continued in a pattern, can comprise love), their experiences of dismissing others and being dismissed (which, continued in a pattern, can indicate bullying), and/or their experiences of separateness from the other. In particular, we would be interested to see how these experiences connect to characteristically mathematical communications.

It is important to emphasize again that all of the incidents described above come from the same class. Whatever the teacher and school culture (including other students) did before the day of these mathematical interactions facilitated interactions of responsiveness, interactions of dismissiveness, and reclusion. We argue that what happens in a classroom cannot be laid at the feet of the teacher or school administration alone. There are larger discourses of school and of school mathematics that strongly influence the students' perceptions of what they could do and what they should do in mathematics class. And these discourses also significantly direct and constrain the teacher and other school personnel.

Wherever we have worked, in a range of countries and jurisdictions within countries, it has been clear that educators want to promote collaboration and student cooperation. At the same time political forces demand competition, which may range from large-scale international comparisons to local grading in classrooms, and these forces constrain the work of educators. There has been some work in mathematics education that describes supportive environments for learning mathematics (e.g., Boaler, 2015), and work that shows that students need to learn skills for collaborating and cooperative learning (e.g., Andersson, 2011). Nevertheless, our research here shows how endemic the relational aspects of student experience are to common mathematical discourse practices. If communication in mathematics classrooms can include love and bullying, it behoves us as researchers and teachers to work to understand better how this works.

We close with some thoughts about love, which arise from our analysis. It may be obvious that we favour loving interactions, in line with the thoughts of, for example, Leggo (2011). Responsiveness is central to love, but our data and analysis reminded us that there is a range of ways to be responsive and a range of people or concerns to which one can be responsive. Responsiveness to a mathematical task (perhaps the love of mathematics) may eclipse responsiveness to the needs of a working partner (the love of a person). And responsiveness to one person may mean ignoring another, at least temporarily. Nevertheless, responsiveness to mathematics alongside and in alignment with a partner can also be an act of love. Likewise, responsiveness to all one's group members does not require a competition of love as if it were a finite resource. The framework we developed for identifying responsiveness and dismissiveness, and thus for reflecting on love and bullying in interactions, helped us see the complexity of the nature of responsiveness and the nature of human relationship. The complexity of responsiveness underscores the tension between personal

autonomy and discursive demands, a necessary tension which Aoki (1999) described in terms of curriculum-as-lived and curriculum-as-planned: "a call to recognize that textured site of lived tension—so often ambiguous, uncertain, and difficult" (p. 181). Honouring this call, we think that a framework that does not draw neat lines of demarcation is most appropriate for consideration of something so complex as love and its antonyms.

We are seeing more clearly now that love requires tension. Love is responsiveness to one's own needs, the needs of others and the "needs" of whatever discourse that is present, all of which exist in tension. Thus, love requires uncertainty, vulnerability and intimacy. A good mathematics teacher will be aware of these tensions and help students navigate them. We believe that such masterful teaching will not only help students to be more loving people, it will also facilitate the development of good mathematics understanding. While we saw glimpses of the good mathematics correlating with loving interactions in our data in the classroom described above, we hope to do and read about further research that helps us see how this works.

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